GEOMAGNETIC FIELD CHARACTERISTICS REVEALED BY EMPIRICAL ORTHOGONAL FUNCTIONS

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Using COV-OBS.x1 (Gillet *et al.*, 2015) main geomagnetic field model, covering the time span 1840–2020, we decomposed the geomagnetic field at Earth's surface, namely the vertical and horizontal components, in oscillation modes at various timescales by means of empirical orthogonal functions (EOF). The EOF analysis shows that the first three oscillation modes of both components are characterized by periodicities of >100 years while modes 4 and 5 are characterized by dominant periodicities of 60–90 years. The first three EOF modes are responsible for more than 90% of the observed features of the vertical and horizontal components at Earth's surface.

Key words: Earth's surface geomagnetic field, empirical orthogonal functions, oscillation modes.

1. INTRODUCTION

The mathematical background of the method was introduced in social science studies carried out by Pearson (1902) and Hotelling (1933); the latter name it principal component analysis (PCA). Lorenz (1956) used the EOF method (empirical orthogonal functions) in a forecasting study of the sea-level pressure field over the United States and southern Canada setting the ground to its application in weather science. Bjornsson and Venegas (1997) were the first to discuss the confusion in literature between the Principal component analysis (PCA) and the Empirical orthogonal functions (EOF) as both terms were used to describe the same method. The EOF/PCA is a versatile method that permits among others dimensionality reduction of large data sets and pattern extraction. The EOF/PCA allows defining the spatial distribution of the variability modes of a field, their temporal variation as well as their relevance in terms of variance (Bjornsson, Venegas, 1997).

The EOF/PCA method, widely used in meteorology and oceanography, is the subject of numerous chapters in books (von Storch, Navarra, 1995; von Storch, Zwiers, 1999; Wilks, 2011) and books (Preisendorfer, 1988; Jolliffe, 2002; Hannachi, 2004; Navarra, Simoncini, 2010).

In geomagnetism, Pais *et al.* (2015) used the PCA on the core flows inverted from geomagnetic field models covering long time spans (*gufm1* and COV-OBS) and showed that mode 1 highlights three large vortices at medium and high latitudes that have opposite circulation under the Atlantic and the Pacific hemispheres, mode 2 carries most of the variations of the Earth's core angular momentum and has a quasi-periodicity of 80 to 90 years, while mode 3 has a more complex spatial pattern showing a distribution of smaller-scale vortices. More recently, Domingos *et al.* (2019) applied PCA to decompose the internal geomagnetic field from the CHAMP (2001–2009) and Swarm (2014–2017) satellite records showing that the spatial structure of the first mode yields the average secular variation (SV) for the time span of the used data and mode 2 is attributed to the secular acceleration (SA).

In this paper we attempt to decompose the vertical and horizontal components of the geomagnetic field at Earth's surface at various timescales using Empirical orthogonal functions (EOF). The terms EOF and PC (principal component) will be used in this paper according to Bjornsson and Venegas (1997), namely the term EOF will be used to describe the spatial distribution of the oscillation modes of the geomagnetic field while the term PC will be used for the temporal variation of the oscillation modes.

2. DATA AND METHOD

The COV-OBS.x1 (Gillet *et al.*, 2015) main geomagnetic field model, covering the time span 1840–2020, was used to obtain time series of the vertical (Z) and of the horizontal (H) components at the Earth's surface in a 2.5° × 2.5° latitude/ longitude grid. The code and coefficients of the model are available online at http://www.spacecenter.dk/files/magnetic-models/COV-OBSx1/.

Following Hannachi (2004) we describe the steps needed in order to obtain EOFs. The gridded data defined as **X**(t, s), **X** being the geomagnetic field at time t_i, for $i = 1, \ldots$ n, and grid point s_j, for $j = 1, \ldots$ m, is then arranged as follows

$$\mathbf{X} = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{pmatrix} \quad (1)$$

where the columns of the matrix describe values at the same grid point.

Further, the data matrix needs to be transformed so as to have zero mean vector. Hence, we need to subtract the mean value of each corresponding row of **X** in the manner $\mathbf{X} = \mathrm{Xraw} - \overline{\mathrm{x}}$, where $\overline{\mathrm{x}}$ is the vector of sample means that can be written

$$\overline{\mathrm{x}} = \frac{1}{\mathrm{n}} \mathbf{1}_{\mathrm{n}}^{\mathrm{T}} \mathbf{X} \quad (2)$$

with $\mathbf{1}_n = (1, \ldots 1)^{\mathrm{T}}$ being the column vector containing only ones.

Having the data scaled we can then write the covariance matrix **S** as follows

$$\mathbf{S} = \frac{1}{\mathrm{n}} \mathbf{X}^{\mathbf{T}} \mathbf{X} \quad (3)$$

that contains the covariances between the time series of the geomagnetic field at any pair of grid points.

Further we use the singular value decomposition (SVD) method in order to get the EOF and PC of data. The gridded data matrix **X** can be decomposed using the SVD:

$$\mathbf{X} = \mathrm{A}\,\Lambda\,\mathrm{U}^{\mathrm{T}} \quad (4)$$

A and U are unitary matrices $\mathrm{U}^{\mathrm{T}}\,\mathrm{U} = \mathrm{A}^{\mathrm{T}}\,\mathrm{A} = \mathrm{I}_q$, where $q \leq \min(n, m)$ is the rank of **X** and I_q is the identity matrix of order q. The matrix Λ is diagonal, the diagonal elements of it are the singular values of **X**. The columns of A and U are the left (PCs), respectively the right (EOFs) singular vectors of the data matrix **X**.

After we use the decomposition in singular values of **X** the covariance matrix **S** can be written as follows:

$$\mathbf{S} = \mathrm{U}\,\Lambda^2\,\mathrm{U}^{\mathrm{T}} \quad (5)$$

where the singular values of the diagonal matrix Λ^2 are ordered in decreasing order; the total variance is given by the diagonal elements of Λ^2. The EOFs and PCs are the right, respectively left singular vectors of the scaled data matrix **X**.

We apply the EOF method to the Z and H time series, in a 2.5° × 2.5° global grid, from COV-OBS.x1 in order to decompose the geomagnetic field at Earth's surface in oscillation modes at various timescales.

3. RESULTS AND DISCUSSION

The EOF analysis shows that the first three oscillation modes (Fig. 1), in both components, are characterized by dominant periodicities of >100 years. The first mode describes 75.83% of the characteristics of the vertical component at Earth's while the contribution of modes 2 and 3 is 13.29%, respectively 4.92%. In case of the horizontal component of the geomagnetic field 77.28% of its features are embedded in mode 1, 9.46% in mode 2 and 5.74% in mode 3. The first EOF mode of the vertical component resemble to some extent the first mode in CHAMP and Swarm radial field data of Domingos *et al.* (2019). However, the periodicities associated with these modes are different as we use in our study data spanning 180 years (1840–2020) in comparison to 8 years of CHAMP, respectively 44 months of Swarm data used by Domingos *et al.* (2019).

EOF modes 4 and 5 of Z and H (Fig. 2) are characterized by dominant periodicities of 60–90 years. These modes are responsible for 2.91% (mode 4) and 1.86% (mode 5) of the vertical component characteristics, respectively 3.67% (mode 4) and 2.22% (mode 5) of the H characteristics. Although the variance of the modes 4 and 5 is rather small compared to that of the first three modes, these two modes are responsible for the detailed structure of the geomagnetic field. Modes 6–10 that have a total variance of 1.18% (Z) and 1.63% (H) also lend to the detailed structure of the geomagnetic field.

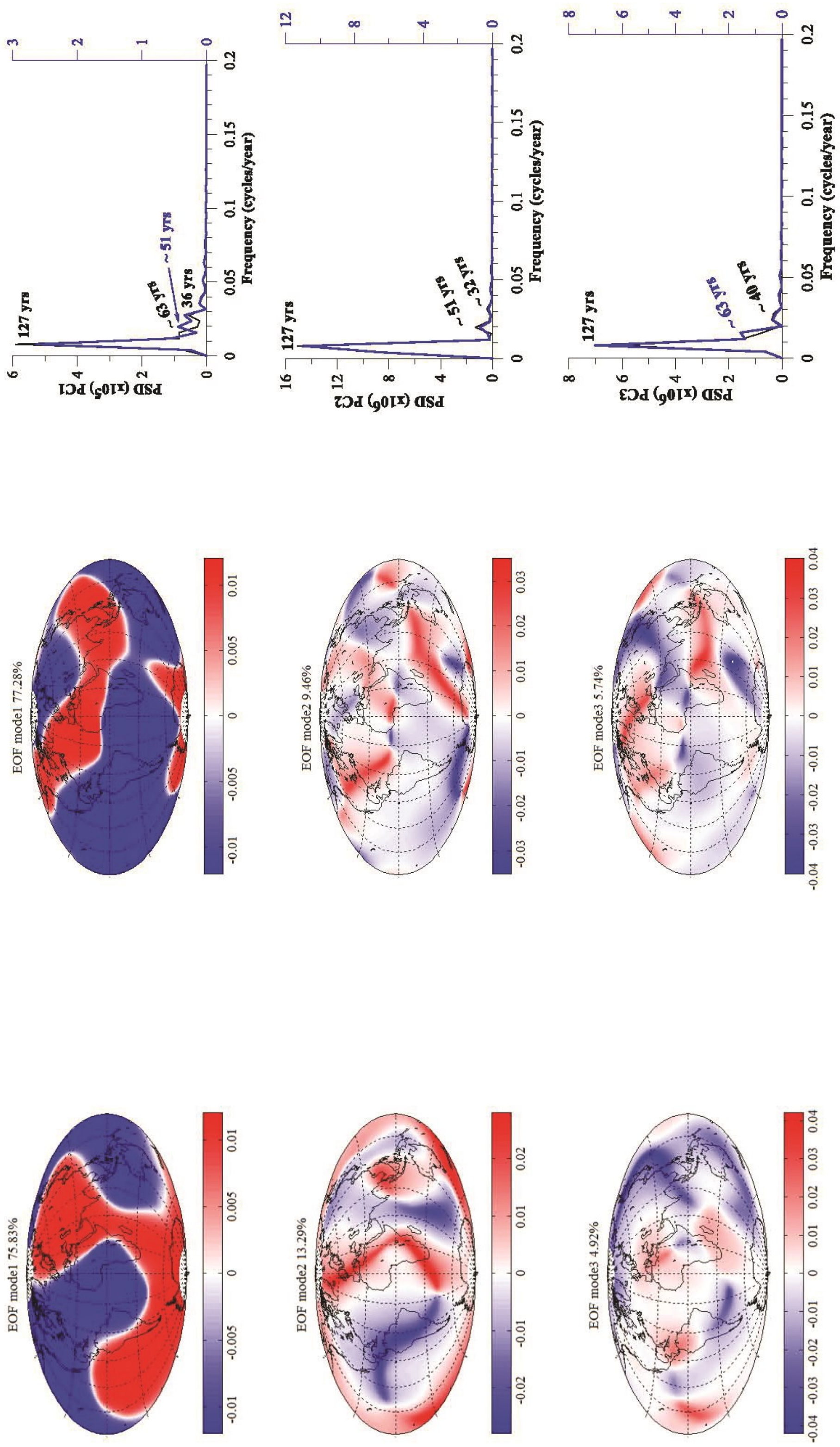

Fig. 1 – EOF modes 1, 2 and 3 in case of the vertical component (left), respectively horizontal component (middle) and their associated frequencies (right – black Z, blue H).

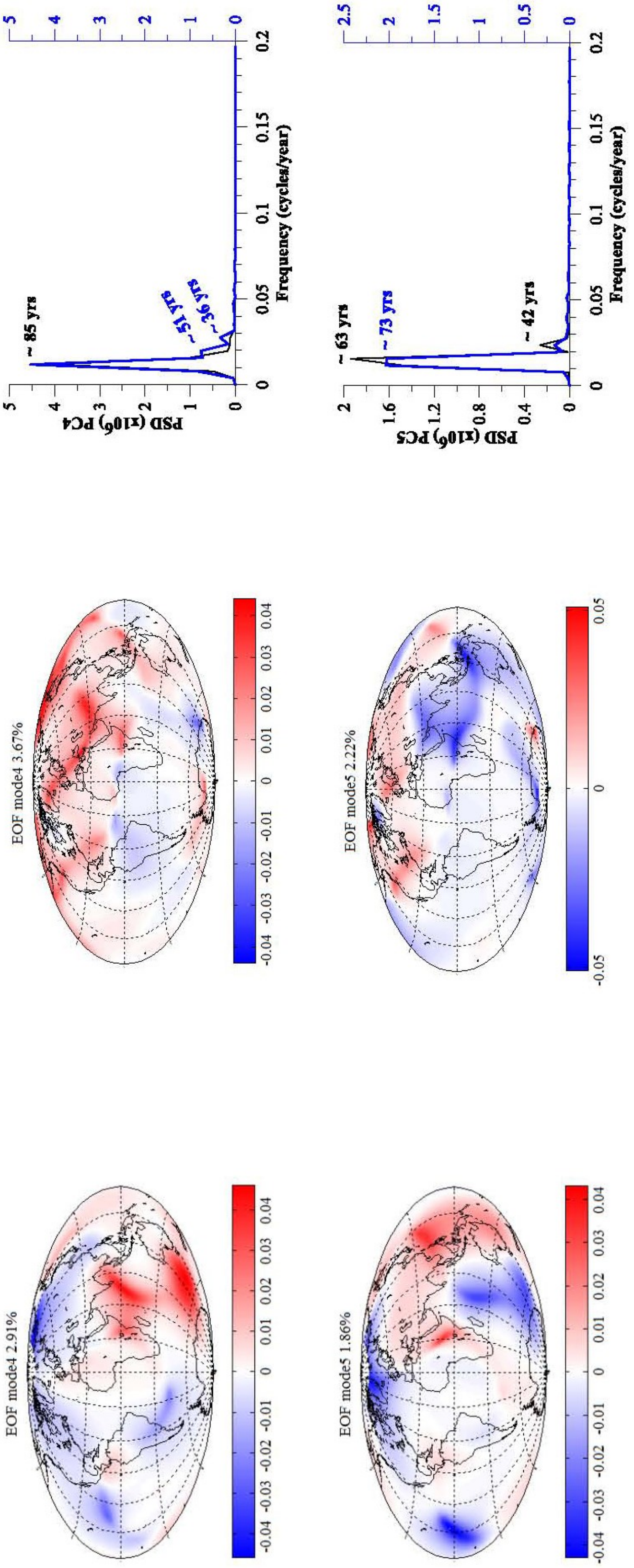

Fig. 2 – EOF modes 4 and 5 in case of the vertical component (left), respectively horizontal component (middle) and their associated frequencies (right – black Z, blue H).

The spatial structures of some of the EOF modes, in case of the vertical component, resemble the geographic distribution of our previous geomagnetic field constituents, at certain epochs, at inter-centennial (>100 years) and sub-centennial (60–90 years) timescales obtained by Hodrick–Prescott, HP, (Hodrick, Prescott, 1997) and Butterworth (1930) filtering approach (Demetrescu, Dobrică, 2014; Ştefan *et al.*, 2017; Dobrică *et al.*, 2018). Further analysis is required in order to see how the EOF modes obtained in this paper are combining in order to explain oscillations at various timescales (inter-centennial and sub-centennial).

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