# APPLICATION OF THE RADON TRANSFORM TO THE STUDY OF TRAVELING SPEEDS OF CORE GEOMAGNETIC FIELD FEATURES. CASE STUDY – THE ~80-YEAR VARIATION

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In this paper we focus on the core surface radial component of the ~80-year variation in the gufm1 model. Time-Longitude (TL) plots at various latitudes between 60N and 60S where constructed in order to gain information regarding the characteristics of the ~80-year variation at the top of the core. A clear westward movement of the ~80-year features from the 30S-30N latitude band is shown. The travelling speeds of the latter are derived by means of the Radon transform.

*Key words:* Radon transform, main geomagnetic field, core-mantle boundary, azimuthally traveling speeds.

## 1. INTRODUCTION

In 1917 the Austrian mathematician Johann Radon introduced the Radon transform, a method for determination of functions from their integrals along straight lines. Methods based on Radon transform are used in different scientific domains. The inverse Radon transform is widely applied in medicine in order to reconstruct the image obtained by X-ray imaging, computerized tomography scans or magnetic resonance imaging (Epstein, 2008). In astronomy it is used to reconstruct the images obtained using a rotating Slit Aperture Telescope and for detection of satellite tracks (Starck and Murtagh, 2002). In geophysics, the inverted Radon transform is the final step in the computation of synthetic seismograms in the analysis of seismic reflection and refraction (Chapman, 1978, 1981). In oceanography Radon transform analysis of TL plots of sea surface height is used to determine the westward propagating speeds of planetary waves (Chelton and Schlax, 1996; Chelton *et al.*, 2003; Cipollini *et al.*, 1997, 2006). Finlay and Jackson (2003), Finlay (2005), and Jackson and Finlay (2007) applied a method based on the Radon transform to study the characteristics of the radial geomagnetic field at the core-mantle boundary. Dobrică *et al.* (2014) used the Radon transform to obtain information on the geomagnetic evolution of secular variation features over the globe.

In the present paper we use the Radon transform method in an attempt to infer information regarding traveling speeds of the radial component features of Earth's magnetic field at the core surface after eliminating high period oscillations from data. Of interest for this paper is the so called ~80-year variation, shown to exist in the surface data by Demetrescu and Dobrică (2005, 2014).

## 2. DATA AND METHOD

The *gufm1* (Jackson *et al.*, 2000) main geomagnetic field model covering the time span 1590–1990, has been used to obtain time series of the radial component at the core surface, $Z_c$, in a 2.5° latitude-longitude grid. The code and coefficients of the model are available online at http://www.epm.geophys.ethz.ch/~cfinlay/. The time series where then treated by means of successive running averages (Demetrescu and Dobrică, 2005, 2014). Smoothed times series, denoted as $Z_c11$, $Z_c22$ and $Z_c78$, result. The so-called ~80-year variation, whose characteristics make the subject of this paper, will be given by the difference $Z_c22$-$Z_c78$. Fig. 1 is a snapshot of the geographical distribution of the core surface radial component of the ~80-year variation for the year 1940.

The first step in order to study the characteristics of the radial component at the surface of the core, both in time and space, was to construct TL plots (Hovmöller, 1949) for different latitudes in steps of 2.5 degrees. In Fig. 2 we show such plots for selected latitudes. As it is a rather difficult task to calculate the traveling speeds of all the field features from all the TL plots, we turned to the previous studies in geomagnetism (Finlay and Jackson, 2003; Finlay, 2005; Jackson and Finlay, 2007) and were able to use a faster method, the Radon transform, to do the job.

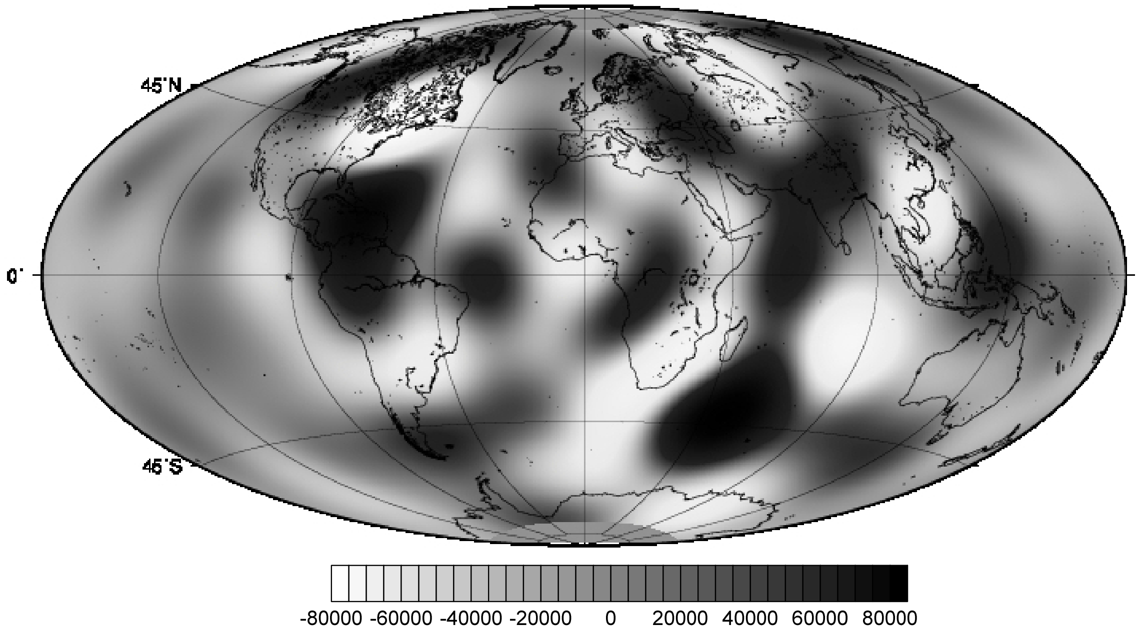


Fig. 1 – Geographical distribution of the core surface radial component of the ~80-year variation (snapshot for 1940).

On short, following Finlay (2005), the Radon transform of a TL plot is defined as the integral over a straight line rotated with an angle *q*:

$$H_R(q,z) = \int_u H(x,t)\Big|_{\substack{x=z\cos q-u\sin q\\ t=z\sin q+u\cos q}} du\,,$$

where $H_R(q,z)$ is the Radon transform of the image $H(x,t)$, $(x,t)$ represents the initial axes and $(q,u)$ are the new axes resulted after rotating the image clockwise by an angle *q*. For every *q* we obtain a sine curve, the intersection point of all sines corresponds to the characteristics from the initial image. We choose the value of *q* to range between -90 and 90 degrees in steps of 2.5 degrees. The resulted array values, after applying the method to every TL plot, are squared and summed along the *z* direction and then combined in a new array. This new array contains the energy of the image as a function of latitude and angle *q*. The final step is to convert the angles *q* into speeds using the following relation (Finlay, 2005):

$$v_n = \frac{2\pi c}{360}\ \sin\theta\ \ \tan q \frac{\Delta\phi}{\Delta t} kmyr^{-1},$$

where $\theta$ is the colatitude, $c$ is the radius. $\Delta\phi$ and $\Delta t$ are longitude, in degrees, respectively time, in years, corresponding to TL plot grid spacing.

In addition we took into account the correspondence between the values of the radial field and that of the color pixels intensities from the TL plots by means of the following formula:

$$I_{new} = \frac{B_{min} + (B_{max} - B_{min})}{(I_{mx} - I_{mn})\ (I - I_{mn})},$$

where $I_{new}$ is the new image, $B_{min}$ and $B_{max}$ represents the minimum, respectively maximum ~80-year variation values from the TL plots. $I_{mn}$ and $I_{mx}$ are the minimum and maximum values of the color pixels and $I$ is the initial image.

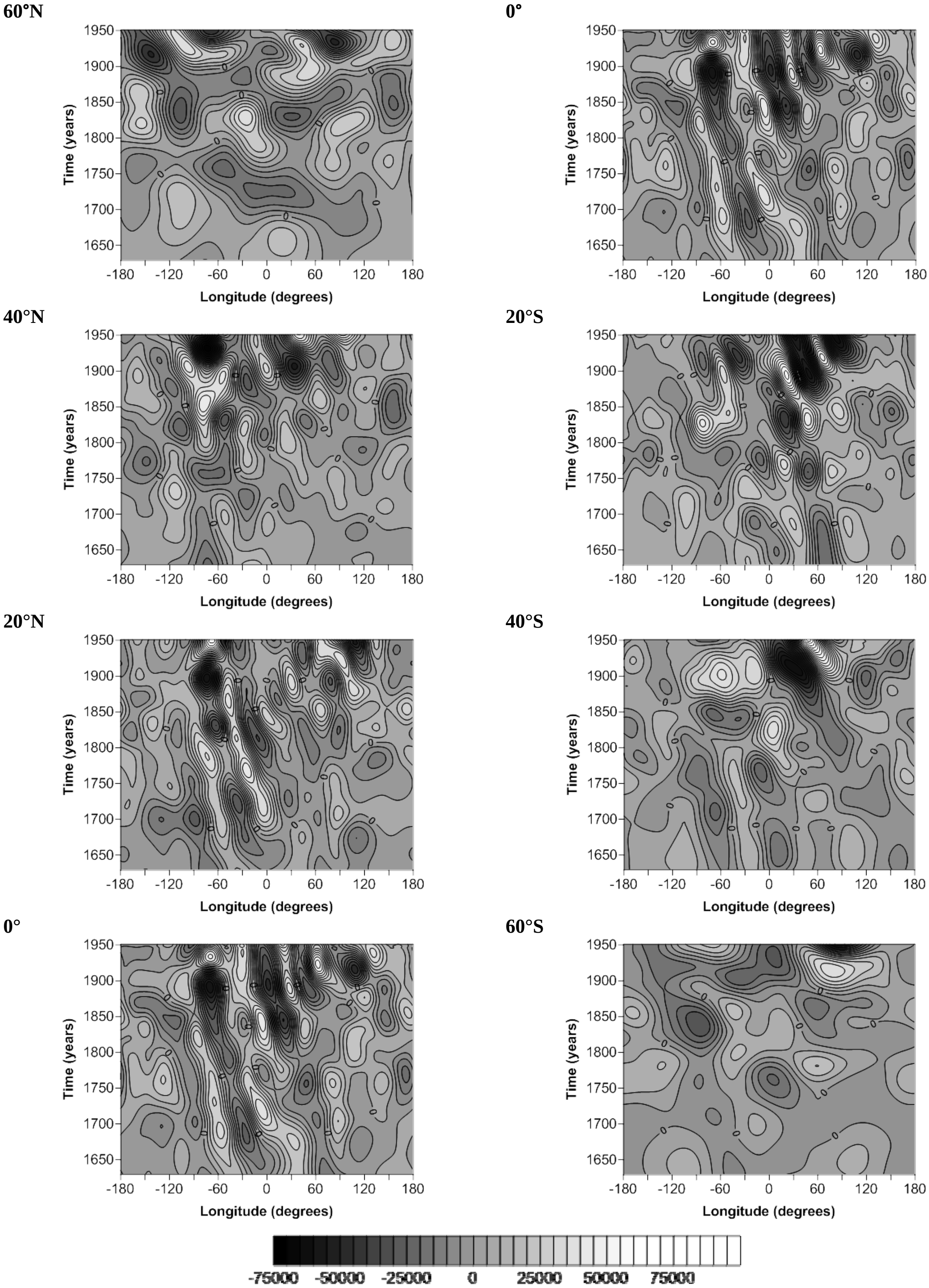


Fig. 2 – Time-Longitude plots of the core surface radial component of the ~80-year variation.

### 3. RESULTS AND DISCUSSION

Analyzing the TL plots, of which several are given in Fig. 2, one notices alternating positive and negative geomagnetic field features with a preferred orientation, meaning in this case westward movements of core field features. The traveling features are well organized in the latitude band ±30 degrees, but the TL plots also show moving features at higher latitudes.

Applying the Radon transform of the TL plots, improved by the calibration of image intensities, we obtain first the energy of the image as a function of latitude and $q$ and then the energy of the traveling speeds of the ~80–year variation features (Fig. 3). In the equatorial band a westward moving dominant peak in the energy density plot can be noticed. It corresponds to a speed of about 17 km/year. At mid latitudes the energy is lower and the traveling speeds reach 35–40 km/year in the southern hemisphere, and 16 km/year in the northern hemisphere. Also, a low energy peak seen in Fig. 3 at positive speeds indicates an eastward movement with a speed of about 10 km/year north of the equator.

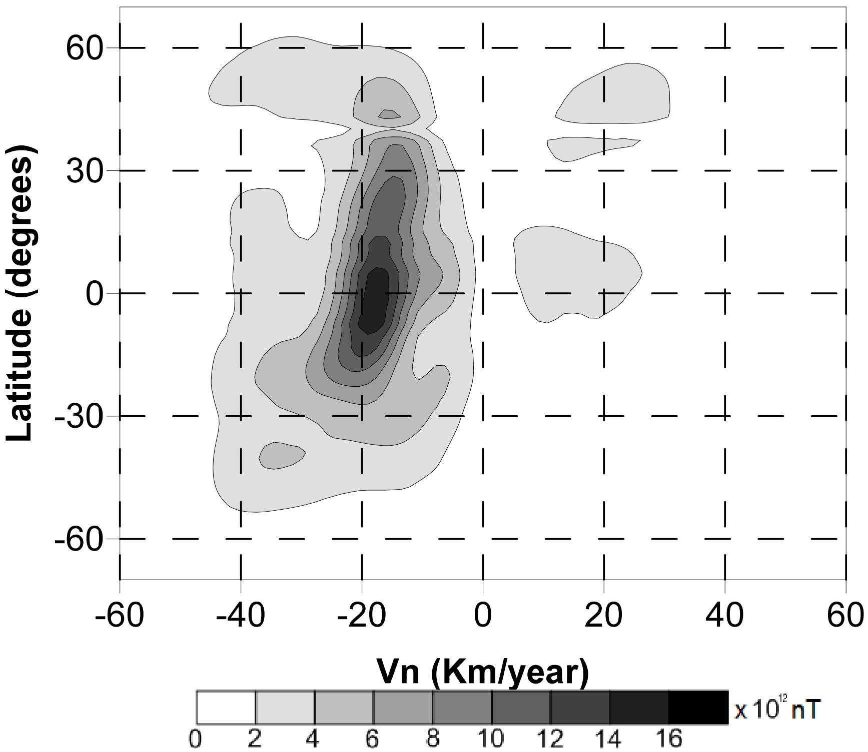


Fig. 3 – Latitude-speed energy plot of the core surface radial component of the ~80-year variation.

Finlay and Jackson (2003) and Jackson and Finlay (2007) observed, after removing from the radial component at the core surface the time-averaged axisymmetric component, high-pass filtered with a cutoff period of 400 years, an equatorially westward motion of about 17 km/year. They conclude that this zonal motion may be accounted for, on one hand, by an equatorial jet or a MAC (magnetic, Archimedes, Coriolis) wave or, on the other hand, by a combination between a jet and a MAC wave. The main result of the present paper, namely the zonal motion of the ~80-year variation radial component with a similar speed, shows that the window of 400 years characterizing the wave of Finlay and Jackson can be narrowed to 80 years. This would result in better constraining the temporal evolution of the core field in geodynamo modeling.

## 4. CONCLUSIONS

Using a filtering approach that successively eliminates (quasi)oscillatory behaviors of decreasing frequency present in the radial field at core surface, it was shown that a (quasi) oscillation of about 80-years dominates the time evolution of the core surface geomagnetic field.

The Radon transform applied to the TL plots constructed for different latitudes indicates a westward movement of the core surface features of the ~80-year variation with a dominant traveling speed of about 0.27 degrees/year (~17 km/yr). The Radon transform has been improved by calibrating the TL image by taking into account the correspondence between the values of the radial field and that of the color pixels intensities.

The results show that the core surface features and their evolution, obtained by Finlay and Jackson (2003) and Finlay and Jackson (2007) high-pass filtering the data with a 400-year cut-off window after removing time-averaged axisymmetric component of the core radial field, is completely explained by the ~80-year variation. The narrowing of the window from 400 years to 80 years would result in better constraining the temporal evolution of the core field in geodynamo modeling.

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