ON SPECTRAL EVALUATION OF MICROSEISMIC SIGNALS*

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L'évaluation spectrale des signaux microséismiques. Les signaux microsismiques sont considérés les oscillations de faible intensité du sous-sol qui sont induites naturellement et qui peuvent être enregistrées à la surface du sol à n'importe quel moment. Pour étudier la structure et les paramètres mécaniques du sous-sol, les signaux microsismiques sont d'habitude analysés d'après leur spectre d'amplitude dans un domaine donné des fréquences. Ainsi, l'évaluation spectrale des signaux microsismiques qui sont enrégistrés dans un certain point du sol s'avère une opération importante dans la prospection sismique. On présente une méthodologie de stabilisation statistique et d'évaluation numérique du spectre de fréquence microsismique par l'utilisation d'un algorithme de décomposition DFT (Discrete Fourier Transform) qui peut réduire le nombre des échantillons de fréquence par rapport au nombre d'échantillons de temps des signaux enrégistrés. On présente la déduction analytique de la méthodologie et l'application à un exemple aux données réelles.

Key words: microseismic survey, signal, amplitude spectrum, statistical and numerical evaluation.

INTRODUCTION

Gorbatikov *et al.* (2004) presented a number of experimental results to demonstrate the application of the microseismic sounding method to a number of typical geological problems in the oil and gas industry and diverse geological structures. The distribution of microseism amplitudes was measured for various frequencies in the range between several hundredth parts of Hz and several Hz in reference areas located on the surface just above the investigated structures. Each case revealed that the structures with higher seismic velocities appeared on the surface as zones with decreased values of spectral amplitudes, whereas structures with lower seismic velocities appeared as areas with increased spectral amplitudes. This method of microseismic sounding implies calculation and observation of spectral relations between the reference and studied site.

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Other microseismic methods include the direct interpretation of the Fourier spectra (Katz and Bellon, 1978) or the determination of spectral relations between horizontal and vertical spatial components (Nakamura, 1989). For this reason, the spectral evaluation is a very important operation in processing of microseismic signals. Using a digital signal processing technique known as "Periodogram" aimed at the estimation of the power spectral density of a time series (see, for example, Oppheneim and Schafer, 1975), we describe a methodology for the statistical stabilization and numerical evaluation of microseismic spectra.

EVALUATION AND STABILIZATION OF MICROSEISMIC SPECTRA

A microseismic signal recorded at a point on the Earth's surface can be considered as a stationary random process. A random process is represented by a function of two real parameters $s_k(t)$, where k takes all the values of integer numbers and t takes all the values on the time axis. For $t = t_1$ we obtain $s_k(t_1)$ which is a random variable defined on the space of k integers. For a fixed value of k, $s_k(t)$ is a deterministic signal that represents "a particular realization" of the random process. A particular realization $s_k(t)$ of the random process is not a signal of finite energy; therefore it does not have a spectrum. For this reason we consider the truncated particular realization $s_{k,T}(t)$ equal to $s_k(t)$ for $|t| \le T/2$ and equal to zero for $|t| > T/2$. The Parseval theorem for the deterministic signal $s_{k,T}(t)$ can be written

$$P_{k,T} = \frac{1}{T}\int_{-T/2}^{T/2}[s_k(t)]^2 dt = \frac{1}{T}\int_{-\infty}^{\infty}[s_{k,T}(t)]^2 dt = \int_{-\infty}^{\infty}\frac{|S_{k,T}(f)|^2}{T}df, \qquad (1)$$

where $P_{k,T}$ represents the power of the signal $s_{k,T}(t)$ and the function $|S_{k,T}(f)|^2/T$ represents a spectral characterization of this truncated particular realization. But for the derivation of a spectral characterization of the random process in its ensemble it is necessary to make a statistical average. For this reason we define the average power of the truncated random process as being the statistical average (relating to parameter k) of powers $P_{k,T}$ by

$$P_{K,T} = \overline{P_{k,T}} = \overline{\int_{-\infty}^{\infty}\frac{|S_{k,T}(f)|^2}{T}df} = \int_{-\infty}^{\infty}\frac{\overline{|S_{k,T}(f)|^2}}{T}df. \qquad (2)$$

For the elimination of the effect of truncation we define the average power of the random process as being

$$P_K = \lim_{T\to\infty} P_{K,T} = \int_{-\infty}^{\infty} \lim_{T\to\infty} \frac{\overline{\left|S_{k,T}(f)\right|^2}}{T} df = \int_{-\infty}^{\infty} q(f)df, \tag{3}$$

where $q(f)$ is the spectral density of power of the random process. This means that for T we have to take a large enough value.

If the random process is stationary, then in the above equations we can consider the interval [0, T) instead of the interval (-T/2, T/2].

A microseismic signal recorded at a point on the Earth's surface can be described as a stationary random process whose particular realizations $s_k(t) \approx s_{k,T}(t)$ are microseismic signal fragments recorded on the intervals [(k-1)T, kT) with T large enough and k equal to 1, 2,..,K.

For the evaluation of the amplitude spectrum of the random process which represents the microseismic signal we compute the Fourier transforms of its particular realizations by the formula

$$S_{k,T}(f) = \int_0^T s_{k,T}(t)e^{-i2\pi ft}dt. \tag{4}$$

Then we stack the K power spectra and obtain the average of these power spectra

$$\left|S_{K,T}(f)\right|^2 = \overline{\left|S_{k,T}(f)\right|^2} = \frac{1}{K}\sum_{k=1}^{K}\left|S_{k,T}(f)\right|^2. \tag{5}$$

The microseismic amplitude spectrum, which will be analyzed for the investigation of geological structures, is

$$\left|S(f)\right| = \sqrt{\left|S_{K,T}(f)\right|^2 / T} = \left|S_{K,T}(f)\right| / \sqrt{T} \approx \sqrt{q(f)}. \tag{6}$$

The minimum value of K is determined by a statistical stabilization of the average power of the random process.

With the formula

$$P_{K,T} = 2\int_{f_1}^{f_2} \frac{\left|S_{K,T}(f)\right|^2}{T} df \tag{2'}$$

we can compute the average power $P_{K,T}$. The frequencies f_1 and f_2 in this formula define the band region of the useful microseisms, which usually are represented by Rayleigh waves. We consider that K is the minimum value for which the average power $P_{K,T}$ is relatively constant (stabilized) if for every K'>K we have

$$\left|P_{K',T} - P_{K,T}\right| < \varepsilon_1, \tag{7}$$

where ε_1 is an accepted error. Certainly, $P_K \approx P_{K,T}$ if T is large enough.

NUMERICAL EVALUATION OF SPECTRA FOR MICROSEISMIC FRAGMENTS OF LARGE LENGTH

A microseismic signal fragment has a very large length. For numerical evaluation of its Fourier transform (see the equation (4)) we use the decomposition DFT algorithm (Sorensen, Burrus, 1993) which uses the fact that fewer frequency samples than time samples are needed. We consider that the signal fragment $s_{k,T}(t)$ is represented by M time samples and its frequency spectrum $S_{k,T}(f)$ is represented by N samples, where $M = RN$ with $N = 2^p$.

For numerical spectral evaluation of a band-limited W signal fragment $s_{k,T}(t)$ we use the Shannon sampling theorem (see, for example, Jurry, 1977) and the decomposition DFT algorithm to obtain

$$S_{k,T}(n\Delta f_N) = \Delta t \sum_{r=0}^{R-1} S_{k,T}^{r}(n\Delta f_N)\,, \quad \text{for n=0,1,....,N/2,,....,N-2, N-1,} \qquad (4')$$

where

$$\Delta t = 1/(2W),\ \ \Delta f_N = 2W/N = 1/(N\Delta t) \qquad (8)$$

and

$$S_{k,T}^{r}(n\Delta f_N) = \sum_{m=0}^{N-1} s_{k,T}((rN+m)\Delta t)e^{-i2\pi\frac{mn}{N}}\,, \text{ for n=0,1,…, N/2,…, N-1,} \qquad (9)$$

which is the DFT of the time series $s_{k,T}(rN\Delta t)$, $s_{k,T}((rN+1)\Delta t)$, $s_{k,T}((rN+2)\Delta t)$,..., $s_{k,T}((rN+N-1)\Delta t)$. When $N = 2^p$ this DFT can be efficiently evaluated by a FFT algorithm.

Therefore, the N frequency samples of the spectrum $S_{k,T}(f)$ of the signal $s_{k,T}(t)$, which is represented by $M = RN$ time samples, can be obtained in the following way. Using a FFT algorithm with the length of $N = 2^p$ we evaluated the DFTs of the R series of type (9), which are then added up term by term. Following this, every term of the obtained series has to be multiplied by the sampling interval Δt.

Taking into consideration the equations (4), (4'), (5), (8), (9) and the fact that

$$T = M\Delta t \qquad (10)$$

we obtain the discrete form of the equation (6)

$$\left|S(n\Delta f_N)\right| = \sqrt{\frac{1}{K}\sum_{k=1}^{K}\Delta t\left|\sum_{r=0}^{R-1} S_{k,T}^{r}(n\Delta f_N)\right|^2 / M} \quad \text{for n = 0,1,...., N/2-1, N/2.} \qquad (6')$$

In (6') we can see that for the numerical evaluation of the amplitude spectrum of a microseismic signal we have to take into consideration the number of particular realizations K, the sampling interval Δt and the number of samples of particular realizations M.

REAL DATA EXAMPLE

Using a microseismic data set, which was recorded at a point on the Earth's surface in a time interval of over 8 hours with the sampling interval $\Delta t = 1/70 \approx 0.014$ s, we illustrate the methodology for the evaluation of microseismic spectra by the following examples. In Fig. 1 we represent a microseismic signal fragment $s_{k,T}(t)$ recorded on the interval [0,T) with T=3.65 s. In Fig. 2 we show the amplitude spectrum $|S(f)|$ of a microseismic signal fragment $s_{k,T}(t)$. This signal was recorded on the interval [0,T) with T = 585.14 s and the number of samples M = 40960. For the spectral evaluation of this signal we used the decomposition DFT algorithm with N = 4096 and L = 10. The Nyquist frequency is 35 Hz and $\Delta f_N = 70/4096 = 0.017$ Hz. In Fig. 3 we represented the amplitude spectrum $|S(f)|$. It was obtained by stacking K=45 power spectra corresponding to 45 microseismic signal fragments $s_{k,T}(t)$ which are recorded on the intervals [(k-1)T, kT), where k is equal to 1, 2,...., K. The length of a fragment $T = M\Delta t \approx 585.24$ s, with M = LN, L=10 and N = 4096. Certainly, for the computation of the power spectra the decomposition DFT algorithm was used. The Nyquist frequency W=35 Hz and $\Delta f = 70/4096 = 0.017$ Hz. It was considered that the band region of the useful microseisms is defined by f_l=0 Hz and f_2 = 0.769 Hz.

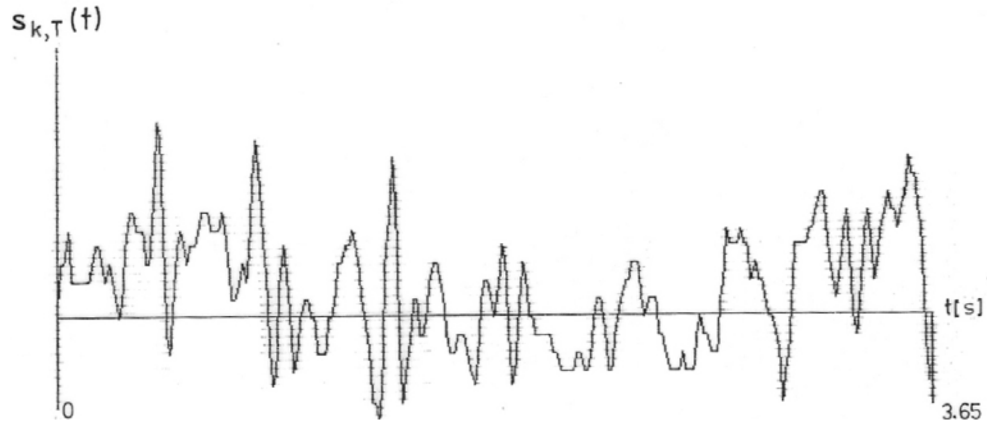

Fig. 1 – A microseismic signal fragment $s_{k,T}(t)$ recorded on the interval [0,T) with T = 3.65 s.

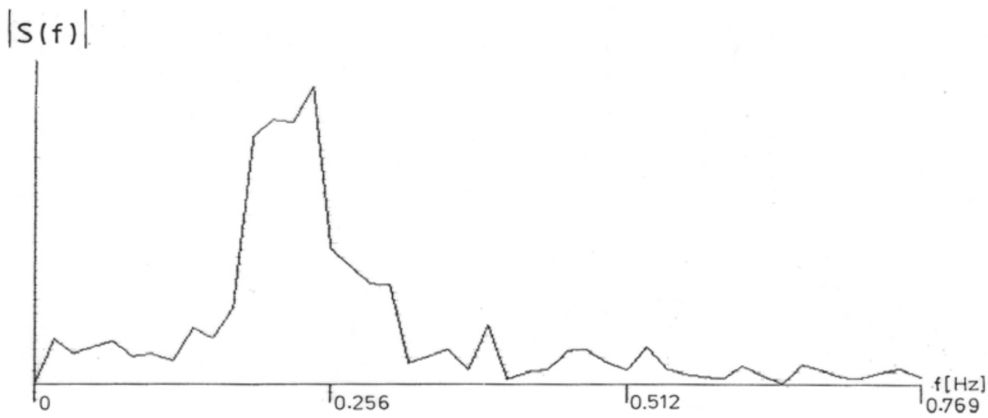

Fig. 2 – The amplitude spectrum $|S(f)|$ of a microseismic signal fragment $s_{k,T}(t)$, which was recorded on the interval [0,T) with T = 585.14 s and with the number of samples M = 40960.

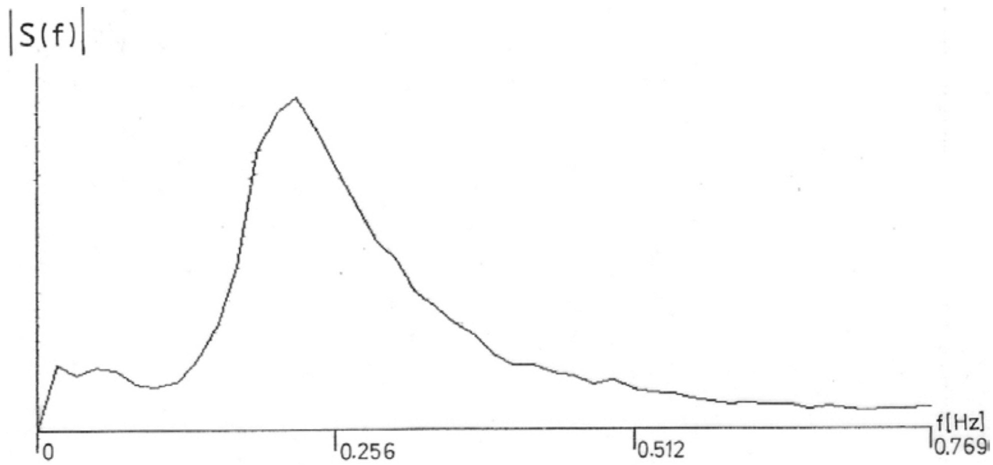

Fig. 3 – The amplitude spectrum $|S(f)|$ obtained by stacking K = 45 power spectra corresponding to 45 microseismic signal fragments $s_{k,T}(t)$ which were recorded on the intervals [(k-1)T, kT], where k is equal to 1, 2,...., K. The length of a fragment $T = M\Delta t \approx 585.24$ s with M = 40960.

CONCLUSIONS

We have considered that a microseismic signal recorded at a point on the Earth's surface can be described as a stationary random process and have presented a methodology for the statistical stabilization and numerical evaluation of its spectrum by stacking power spectra of microseismic signal fragments. Besides showing the analytical derivation of algorithms, we have shown an application to a real data example.

This paper can been also considered as a mathematical background review of microseismic spectral evaluation.

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