SUPERFICIAL AMPLIFICATION EFFECTS INDUCED BY SH WAVES IN NONLINEAR ELASTIC LAYERED HALF-SPACE*

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In order to determine the seismic effects in superficial layers of the earth taking into account their nonlinear constitutive behavior, specific wave equations for longitudinal and transversal propagations are deduced. Following approximations schemes previously enounced by the first author, which furnish effective solutions corresponding to a layered half-space as well as correlations between surface effects (displacements and stresses) and the dislocational mechanism of sources located in ground layer, in the present paper the amplification effect for a layered half-space with nonlinear behavior of the surface layer is analyzed. The results are in good agreement with observational data and consist in specific new effects: the existence of very well defined directivity curves and of almost punctual regions with focusing effects, their dependence on focal mechanism, deep rocks structure and wave superposition, the importance of constitutive laws, the nonlinearity of soft surface soils and of multiple wave overlapped simultaneously with increasing fundamental frequency. These facts are pointing out the correlation between nonlinearity and directivity and are encouraging the efforts for a new approach in viscous-plastic and fissional rocks behavior.

Key words: amplification effects, nonlinear behavior.

1. INTRODUCTION

The nonlinear and viscoelastic behavior of layers at earth surface may sensibly alter the already determined seismic maps which represent the isoseistic lines associated to earthquakes and obtained theoretically only in the frame of a linear elastic analysis. The aimed analysis of this paper is based on the previously developed scheme of approximation of the author (Mişicu, 1953) that according to subsequently published monographic works (Doyle, Ericksen, 1959) presents a very general character. We mention that other different procedures were also elaborated as for instance the one proposed by Signorini (Signorini, 1936).

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Accordingly, the corresponding corrections regard the intensity of seismic effects as well as the associated distribution and directivity. We resort to the following constitutive and dynamic equations

$$\sigma_{ij} = \delta_{ij}\lambda(\theta)\theta + 2\mu(\gamma)\varepsilon_{ij}, \sigma_{ij,j} = \rho\ddot{u}_i, \tag{1}$$

where $\theta=\varepsilon_{ii}$ is the dilatation, $\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i} + u_{k,i}u_{k,j})$ is the strain tensor, u_i are the (Cartesian) components of the deformation vector ($u_{i,j} = \frac{\partial u_i}{\partial x_j}$, $i,j=1,2,3$), $\gamma = \sqrt{\varepsilon^D{}_{ij}\varepsilon^D{}_{ij}}$ is the second-order invariant of the strain deviator, $\varepsilon^D{}_{ij} = \varepsilon_{ij} - \frac{1}{3}\delta_{ij}\theta$, ρ is the density and σ_{ij} is the stress tensor. Accordingly, the second equation from (1) becomes

$$\begin{aligned}&(\lambda+\mu)\theta_{,i} + \mu(\Delta u_i + u_{k,i}\Delta u_k + u_{k,ij}u_{k,j}) + \lambda_{,i}\theta + \mu_{,j}(u_{i,j} + u_{j,i} + u_{k,i}u_{k,j}) = \\ &= \rho\ddot{u}_i; \quad \text{where} \quad \Delta = \nabla^2 = \sum_i \frac{\partial^2}{\partial x_i^2};\end{aligned} \tag{2}$$

The coefficients λ and μ are assumed to be polynomials of, respectively, dilatation and second order invariant of strain

$$\lambda(\theta) = \sum_{n=0}^{2}\lambda_n\theta^n, \quad \mu(\gamma) = \sum_{n=0}^{2}\mu_n(-\gamma)^n \tag{3}$$

A more general Ansatz consists in the assumption that the considered moduli are function of both invariants θ,γ. We also account for the distortion function and the rotation

$$u_{i,j} = \varepsilon_{ij} - \omega_{ij} - \frac{1}{2}u_{k,i}u_{k,j}, \omega_{ij} = \frac{1}{2}(u_{j,i} - u_{i,j}) \tag{4}$$

The replacement of the stress tensor furnished by the first equation from (1) into the second one leads to the differential equation of motion

$$\begin{aligned}&(\lambda_0 + \mu_0)\theta_{1,i} + \mu_0\Delta u_{1i} = \rho\ddot{u}_{1i}, \\ &(\lambda_0 + \mu_0)\theta_{2,i} + \mu_0\Delta u_{2i} + E_{2i} = \rho\ddot{u}_{2i} \\ &(\lambda_0 + \mu_0)\theta_{3,i} + \mu_0\Delta u_{3i} + E_{3i} = \rho\ddot{u}_{3i}, ...\end{aligned} \tag{5}$$

in which occur the terms (the moduli λ_n, μ_n being assumed constant)

$$
\begin{aligned}
&E_{2i} = (\lambda_1\theta_1 + \mu_1\gamma_1)\theta_{1,i} + \mu_0(u_{1k,i}\Delta u_{1k} + u_{1k,ij}u_{1k,j}) + \mu_1\gamma_1\Delta u_{1i} \\
&+\mu_1\gamma_{1,j}(u_{1i,j} + u_{1j,i}) \\
&E_{3i} = (\lambda_1\theta_1 + \mu_1\gamma_1)\theta_{2,i} + (\lambda_1\theta_2 + \lambda_2\theta_1^2 + \mu_1\gamma_2 + \mu_2\gamma_1^2)\theta_{1,i} + \\
&\mu_0(u_{1k,i}\Delta u_{2k} + u_{2k,i}\Delta u_{1k} + u_{1k,ij}u_{2k,j} + u_{2k,ij}u_{1k,j}) + \\
&\mu_1\gamma_1(\Delta u_{2i} + u_{1k,i}\Delta u_{1k} + u_{1k,ij}\Delta u_{1k,j}) + (\mu_1\gamma_2 + \mu_2\gamma_1^2)\Delta u_{1i} + \\
&+\mu_1\gamma_{1,j}(u_{2i,j} + u_{2j,i} + u_{1k,i}u_{1k,j}) + \\
&(\mu_1\gamma_{2,j} + 2\mu_2\gamma_1\gamma_{2,j})(u_{1i,j} + u_{1j,i})
\end{aligned}
\tag{6}
$$

2. APPROXIMATE SOLUTIONS OF WAVE EQUATIONS

Following the approximation schemes exposed in Mişicu (1953) and Signorini (1936) we consider the following expansions

$$u_i = \sum_{n=1,2,..} e^n u_{ni} \tag{7}$$

e being a small parameter (<1). Consequently, the dilatation and strain invariant may be n expanded as follows:

$$
\begin{aligned}
&\theta = \sum_{n=1,..} e^n\theta_n,\ \gamma = \sqrt{\sum_{m,n=1,2,..} e^{m+n}\gamma_{mn}^2} \\
&\theta_n = u_{ni,i},\ \gamma_{mn} = \sqrt{\varepsilon_{mij}\varepsilon_{nij}} = \sqrt{\varepsilon_{m11}\varepsilon_{n11} + \ldots + 2\varepsilon_{m12}\varepsilon_{n12} + \ldots}
\end{aligned}
\tag{8}
$$

fact which involves the additional developments for elastic moduli

$$
\begin{aligned}
&\lambda = \lambda_0 + e\lambda_1\theta_1 + e^2(\lambda_1\theta_2 + \lambda_2\theta_1^2) + \\
&e^3[\lambda_2 2\theta_1\theta_2 + \lambda_3(3\theta_1^2\theta_2 + \theta_1^3)] + \\
&e^4[\lambda_2(\theta_2^2 + 2\theta_3\theta_1) + \lambda_4(\theta_1^4 + 6\theta_1^3\theta_2)] + \ldots
\end{aligned}
\tag{9}
$$

and

$$
\begin{aligned}
&\gamma = e\gamma_{11}\sqrt{1 + 2G.} \\
&G = e\left(\frac{\gamma_{12}}{\gamma_{11}}\right)^2 + e^2\left[\left(\frac{\gamma_{13}}{\gamma_{11}}\right)^2 + \frac{1}{2}\left(\frac{\gamma_{22}}{\gamma_{11}}\right)^2\right] + \ldots
\end{aligned}
\tag{10}
$$

the last one being approximated for $G < 1$ by the function

$$\gamma \cong e\gamma_{11}(1 + G) + \ldots = \sum_{n=1,2,\ldots} e^n\gamma_n \tag{11}$$

for $\gamma_1 = \gamma_{11}, \gamma_2 = \gamma_{11}(\gamma_{12} / \gamma_{11})^2, \ldots$

Accordingly the associated approximated differential equations which may be derived by identifying terms of identical order (Mişicu, in press) were obtained from the equations for dilatational and rotational waves

$$(\lambda_0 + 2\mu_0)\Delta\theta_1 = \rho\ddot{\theta}_1 \tag{12}$$

$$(\lambda_0 + 2\mu_0)\Delta\theta_2 + \Theta_2 = \rho\ddot{u}_{2i}, (\lambda_0 + 2\mu_0)\Delta\theta_3 + \Theta_3 = \rho\ddot{\theta}_3, \ldots$$

and

$$\mu_0\Delta\omega_{1ij} = \rho\ddot{\omega}_{1ij}, \mu_0\Delta\omega_{2ij} + \Omega_{2ij} = \rho\ddot{\omega}_{2ij}, \mu_0\Delta\omega_{3ij} + \Omega_{3ij} = \rho\ddot{\omega}_{3ij}, .. \tag{13}$$

for

$$\begin{aligned} &\Theta_2 = E_{2i,i}, \Theta_3 = E_{3i,i}, \\ &\Omega_{2ij} = \frac{1}{2}(E_{2j,i} - E_{2i,j}), \Omega_{3ij} = \frac{1}{2}(E_{3j,i} - E_{3i,j}) \end{aligned} \tag{14}$$

3. GENERAL NONLINEAR SH PLANE WAVES OF THE FIRST APPROXIMATION

Making use of the linear solutions of the first equation given by (13), which correspond to *SH* waves satisfying the conditions $u_{11} = u_{13} = 0, u_{12} \neq 0$, we get $\omega_{123} = -u_{12,3}/2, \omega_{112} = u_{12,1}/2$, $\omega_{131} = 0$ so that the equation reduces to: $\Delta u_{12} - \beta^{-2}\ddot{u}_{12} = 0$ where $\beta^2 = \mu_0/\rho$. This equation admits the solution:

$$u_{12} = e^{i\xi}[u^{"}e^{i\varsigma} + u'e^{-i\varsigma}], (\xi = \omega t - kx\sin f, \varsigma = kz\cos f) \tag{15}$$

Here $k = \omega/\beta$, f is the incidence angle between the normal to the waves front and the vertical axis and *u*' and *u*" are the coefficients of the linear solutions. We denote by $c = \beta/\sin f$ the horizontal phase velocity. The corresponding deviatoric invariant is

$$\begin{aligned} \gamma_1 &= \sqrt{\frac{1}{4}(u_{12,1}^2 + u_{12,3}^2)} = \\ &= \frac{ik}{2}e^{i\xi}\sqrt{(u''e^{i\varsigma} + u'e^{-i\varsigma})^2\sin^2 f + (u''e^{i\varsigma} - u'e^{-i\varsigma})^2\cos^2 f} \end{aligned} \tag{16}$$

If the waves front includes only ascendent components ($u' \cong 0$) then:

$$\gamma_1 \cong \frac{ik}{2}e^{i(\xi+\varsigma)}u'' \tag{17}$$

whereas for the descendent one ($u'' = 0$):

$$\gamma_1 \cong \frac{ik}{2} e^{i(\xi-\varsigma)} u' \tag{18}$$

For an angle $f \cong 0, \pi$ relation (16) becomes:

$$\gamma_1 = \frac{ik}{2} e^{i\xi} (u'' e^{i\varsigma} - u' e^{-i\varsigma}) \cos f \tag{19}$$

and for $f \cong \frac{\pi}{2}$:

$$\gamma_1 = \frac{ik}{2} e^{i\xi} (u'' e^{i\varsigma} + u' e^{-i\varsigma}) \sin f \tag{20}$$

We stress together the listed relations as:

$$\gamma_1 = \frac{ik}{2} e^{i\xi} (a u'' e^{i\varsigma} + b u' e^{-i\varsigma}) \tag{21}$$

We observe that this relation may be regarded as an independent definition of the invariant which occurs in the constitutive equations and not as an approximated quantity. In this sense the respective fitness has to be checked directly with the experimental data. Besides we also stress the fact that the last relation appears as valid for the above mentioned cases, *i.e.* for regions where only ascendent or descendent waves are important or for front orientations horizontal or vertical. More complete approximations may be yielded as follows. Relation (16) may be put under the form

$$\gamma = \frac{ik}{2} e^{i(\xi+\zeta)} u'' \sqrt{1 + e^{-4i\zeta} (\frac{u'}{u''})^2 - 2e^{-2i\zeta} \frac{u'}{u''} \cos 2f} \tag{22}$$

which emphasizes the fact that in regions where $u' << u''$ (with predominant ascendent waves) we have

$$\gamma \cong \frac{ik}{2} e^{i\xi} u'' \left(1 + \frac{e^{-3i\zeta}}{2} (\frac{u'}{u''})^2 - e^{-i\zeta} \frac{u'}{u''} \cos 2f \right) \tag{23}$$

Analogously for $u' >> u''$ follows the approximation

$$\gamma \cong \frac{ik}{2} e^{i\xi} u' \left(1 + \frac{e^{3i\zeta}}{2} (\frac{u''}{u'})^2 - e^{i\zeta} \frac{u''}{u'} \cos 2f \right)$$

From the applicative point of view ascendent waves correspond to incipient fronts arriving for instance at the earth surface and descendent ones to final fronts after refractions and reflexions at the same surface. Hence we get from the first relation from (6) in the case of purely transversal waves the expressions corresponding to the invariant (16)

$$\begin{aligned}
E_{21} &= \mu_0[u_{12,1}\Delta u_{12} + u_{12,11}u_{12,1} + u_{12,13}u_{12,13}] = \\
&2i\mu_0 k^3 e^{2i\xi}\sin f.[u''^2 e^{2i\varsigma} + u'^2 e^{-2i\varsigma} + 2u''u'\sin^2 f) \\
E_{22} &= \mu_1[\gamma_1\Delta u_{12} + \gamma_{1,1}u_{12,1} + \gamma_{1,3}u_{12,3}] = \\
&i\mu_1 k^3 e^{2i\xi}[au''^2 e^{2i\varsigma} + bu'^2 e^{-2i\varsigma} + (a+b)u''u'\sin^2 f] \\
E_{23} &= \mu_0[u_{12,3}\Delta u_{12} + u_{12,31}u_{12,1} + u_{12,33}u_{12,3}] = \\
&-2i\mu_0 k^3 e^{2i\xi}\cos f.[u''^2 e^{2i\varsigma} - u'^2 e^{-2i\varsigma}]
\end{aligned} \tag{24}$$

Meanwhile the rotational terms from (14) become

$$\begin{aligned}
\Omega_{212} &= \mu_1 k^4 e^{2i\xi}\sin f[au''^2 e^{2i\varsigma} + bu'^2 e^{-2i\varsigma} + (a+b)u''u'\sin^2 f] \\
\Omega_{223} &= \mu_1 k^4 e^{2i\xi}\cos f[au''^2 e^{2i\varsigma} - bu'^2 e^{-2i\varsigma}],\ \Omega_{231} = 0,
\end{aligned} \tag{25}$$

for the wave equations from (13)

$$\begin{aligned}
&\mu_0\Delta\omega_{212} + \Omega_{212} = \rho\ddot{\omega}_{212}, \mu_0\Delta\omega_{223} + \Omega_{223} = \rho\ddot{\omega}_{213}, \\
&\mu_0\Delta\omega_{231} = \rho\ddot{\omega}_{231}
\end{aligned} \tag{26}$$

The last equation is not relevant being reducible to the similar one from (13). We take into account the solution:

$$u_{22} = e^{2i\xi}\{u_2 + u_2'' e^{2i\varsigma} + u_2' e^{-2i\varsigma} + e^{2i\varsigma}(w''_2\,\xi + \alpha''_2\,\zeta) + w'_2\, e^{-2i\varsigma}(w'_2\,\xi + \alpha'_2\,\zeta)] \tag{27}$$

Eq. (27) furnishes the conditions:

$$u_2 = \frac{\mu_1}{4i\mu_0}k(a+b)u''u'tg^2 f, w''_2 = -\frac{\mu_1}{4\mu_0}kau''^2 - \alpha''_2, w''_2 = -\frac{\mu_1}{4\mu_0}kbu'^2 + \alpha'_2 \tag{28}$$

We observe that the effective displacements are $v = v_{11} + v_{12} + \ldots$, $(v_{1n} = e^n u_{1n})$. Taking into account the boundary conditions $u_{22} = u_{22}(z_m)$, $\sigma_{22} = \sigma_{22}(z_m)$ for $z = z_m$, m=1,2,… we obtain the displacements and stresses

$$\begin{aligned}
&u_{22} = u_{22}^0 + \xi u_{22}^1, \sigma_{223} = \sigma_{223}^0 + \xi\sigma_{223}^1 \\
&u_{22}^0 = e^{2i\xi}\{MC + e^{2i\varsigma}(u''_2 + \alpha''_2\,\zeta) + e^{-2i\varsigma}(u'_2 + \alpha'_2\,\zeta)\}, \\
&u_{22}^1 = -e^{2i\xi}\{e^{2i\varsigma}(MA + \alpha''_2) + e^{-2i\varsigma}(MB - \alpha'_2)\}, \\
&\sigma_{223}^0 = \mu_1 e^{2i\xi}2ik\cos f\{e^{2i\varsigma}[u''_2 + \alpha''_2(\zeta + 1/2i)] - e^{-2i\varsigma}[u'_2 + \alpha'_2(\zeta - 1/2i)]\}, \\
&\sigma_{223}^1 = -e^{2i\xi}2ik\cos f\{e^{2i\varsigma}(MA + \alpha''_2) - e^{-2i\varsigma}(MB - \alpha'_2)\},
\end{aligned} \tag{29}$$

for

$$M = \mu_1 k / 4i\mu_0 , A = au''^2 , B = bu'^2 , C = (a+b)u''u' tg^2 f$$

4. WAVE PROPAGATION IN A LAYER WITH FREE SURFACE

We consider a layer $0 \le d \le z$ with a non-loaded surface $z = 0$. Meanwhile at the interface $z = d$ no arriving ascendent front waves with frequency 2ω do occur. For this reason we have to cancel the terms containing $e^{-2i\zeta}$ in the solution (27). Hence we have the conditions:

$$u_{22}^0 = 0, u_{22}^1 = 0 \ \sigma_{223}^0 = 0, \sigma_{223}^1 = 0 \tag{30}$$

We yield the solution:

$$\begin{gathered} u_{22} = Me^{2i\xi}\{C(1-e^{-2ik(d-z)\cos f}) + e^{2i\varsigma} Ak(d-z)\cos f] - \\ e^{-2i\zeta}[(A-B\}/2 + iBk(d-z)\cos f + Ce^{-2ikd\cos f}]\} \end{gathered} \tag{31}$$

In order to obtain the amplification effects amplitudes due to nonlinearity compared with the linear amplitudes we take into account the ratio $K = u_{22} / u''$ for $z = 0$ which expresses the increment of displacement values of ascendent waves at the basis of the superficial layer (corresponding to already amplified effects through the linearly behavior of deeper rocks). We resort to the notations (T stands for the oscillation period).

$$m = \frac{\mu_1}{\mu_0}, \theta = \Delta \cos f, \Delta = \frac{d}{L}, U'' = \frac{u''}{L}, L = \frac{T\beta_0}{2\pi} = \frac{1}{k}, \varepsilon = \frac{u'}{u''} \tag{32}$$

We obtain the expression

$$\begin{gathered} K = \frac{1}{LU''} M\mathrm{e}^{2\mathrm{i}\xi}\{C(1-2\cos 2\theta) + \frac{A-B}{2}\frac{1}{4}(i\theta - \frac{1}{2})\} = \\ = \frac{1}{4} mU''\{a(i\theta - \frac{1}{2}) + \varepsilon(a+b)\mathrm{tg}^2 f(1-2\cos 2\theta) + b\varepsilon^2(i\theta - \frac{1}{2})\} \end{gathered} \tag{33}$$

The last results may be further used for effective calculations taking into account the occurring parameters denoted above. We mention the fact that since the linear theory furnishes the expressions $\varepsilon = 1 - \frac{2D}{1+2D}, D = e^{i\theta}\mathrm{tg}\theta$, fact which reduces the number of parameters. We observe that in the range $0 \le \theta \le 2\pi$ the absolute value of ε does not exceed the unity and for θ close to the value π/4 tends to 0 that according to the condition (20) we may take $A = -B$ with $a = -b = 1$. On the other hand in a range shorter than the interval $-30^\circ \le f \le 30^\circ$ we may take $K \approx \chi K', \chi = \frac{1}{8} mU'', K' = 2\sqrt{1+4\theta^2}$. By taking into account both conditions we observe that these assumptions correspond to the northern part of the Romanian Plain close to the town of Bucharest. We resort to the following ranges for the numerical values of parameters: $0.2 \le m \le 0.8, T \approx 0.5''-1''$, $0.015km \le d \le 0.3km, 0.1km/\sec \le \beta_0 \le 1km/\sec$, $0 \le f \le 30^\circ$, $u'' \approx 0.0003\ km$. The enounced ranges correspond to very soft soils (*m*), a reported oscillation period (*T*) during the major seismic events, the thickness of superficial soft soils in the Bucharest region (*d*), the velocity of transversal waves in superficial very soft layers (β_0), the direction of front waves in the mentioned region (*f*) and the maximal reported amplitude $u'' \approx 30\text{cm}$. Accordingly we have $0km \le L \le 0.01km$, $0 \le \Delta \le 15$. More general results may also be obtained for larger ranges as the quoted ones.

The obtained formula emphasizes following qualitative features of amplification effects:

1. K increases proportionally to the ratio m of the second and first shear moduli, fact which expresses the importance of the nonlinear behavior of soft soils in seismic risk evaluation.

2. K increases proportionally to the amplitudes of ascendent seismic waves fact which emphasizes that seismic risk is enhanced by the nonlinear behavior of soft soils especially for strong earthquakes.

3. K is influenced by the parameter Δ which depends upon the ratio of thickness of the layer *d* to characteristic length *L*. This fact involves a variation of the amplification consisting of its vanishing in so-called shadowed regions concentrically located with respect to the epicenter.

4. K decreases with the epicentral distance (determined by the increase of the orientation angle *f*) of front waves according to a nonlinear law, fact which emphasizes the importance of evaluating the seismic risk in soft soils with nonlinear behavior belonging to the epicentral regions.

5. K increases for lower periods of seismic waves and propagation velocities of transversal waves also according to a nonlinear law (in which occurs the characteristic length which depends upon the mentioned parameters).

6. K increases with the thickness of superficial layers due to the softness effects produced on larger trajectory of waves in the soil, again according to the nonlinear law.

In view of effective numerical calculus were plotted in diagrams the curves yielding, respectively, the parameter χ for the values of the ratio

$m = 0.2 \div 0.8$ and $U'' = 0.01 \div 0.04$ (Fig. 1) and the amplification coefficient K for values of parameters $\chi = 0 \div 0.002$ and $\theta = 0^\circ \div 35^\circ$ (Fig. 2).

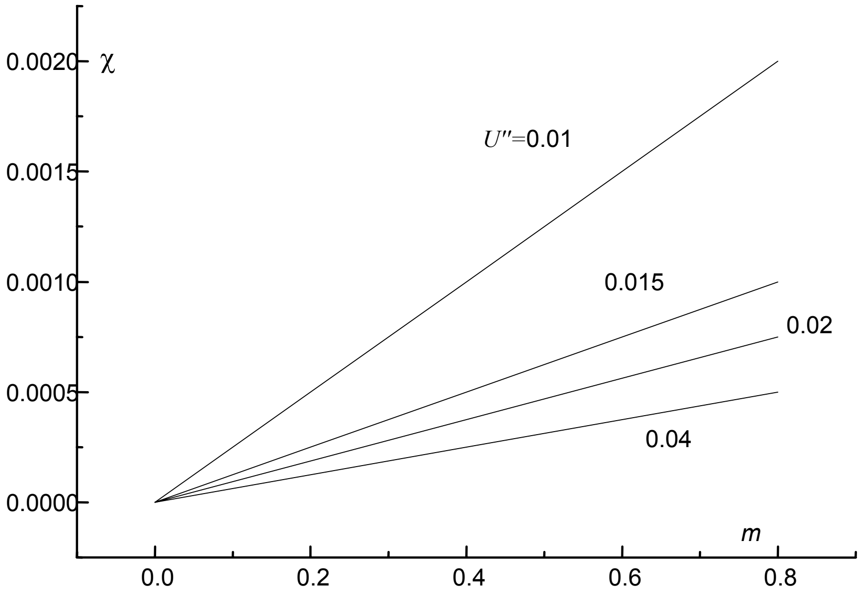

Fig. 1 – Variation diagram $\chi - m$, for $0 \le m \le 0.8, 0.01 \le U'' \le 0.04$.

5. CONCLUSIONS

The maximal additional amplification coefficients due to nonlinear effects (referred to the basic ascendent amplitudes) may reach values corresponding to very soft soils and superficial layers up to 0.28, values appropriate to the amplification due to the linear behavior of layered soil of reported major earthquakes (Cornea *et al.*, 1980; Cornea, Mişicu, 1981), but which were determined without considering the nonlinear branch of stress-strain constitutive curves or the very soft local and very local response of soil. In such cases the entire resulting amplification coefficient may reach a greater value emphasizing a catastrophic exceptional situation. Of course, the above data furnish generally more ponderate indications fit for the mentioned region but do not exclude extremal possibilities since on a large area especially with very altered grounds dues to the human activities such occurrences must not be skipped out.

On the basis of these results and using previous developed isoseistic description of seismic effects extended over the Romanian territory, a new set of maps may be elaborated which furnish the additional amplification effects starting from the older ones obtained by means of linear analysis. This task will be performed in a subsequent work.

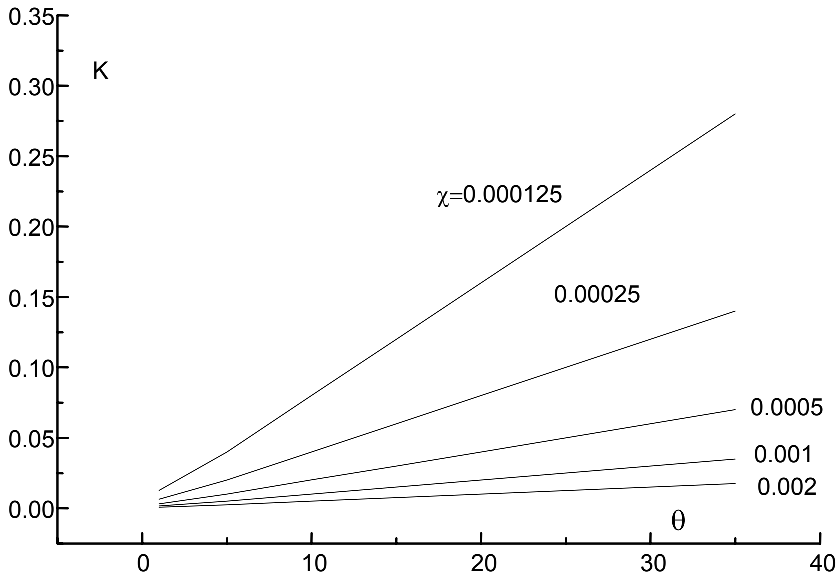

Fig. 2 – Variation diagram for amplification coefficients K for $0.000125 \le \chi \le 0.02$, $0^\circ \le \theta \le 35^\circ$.

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